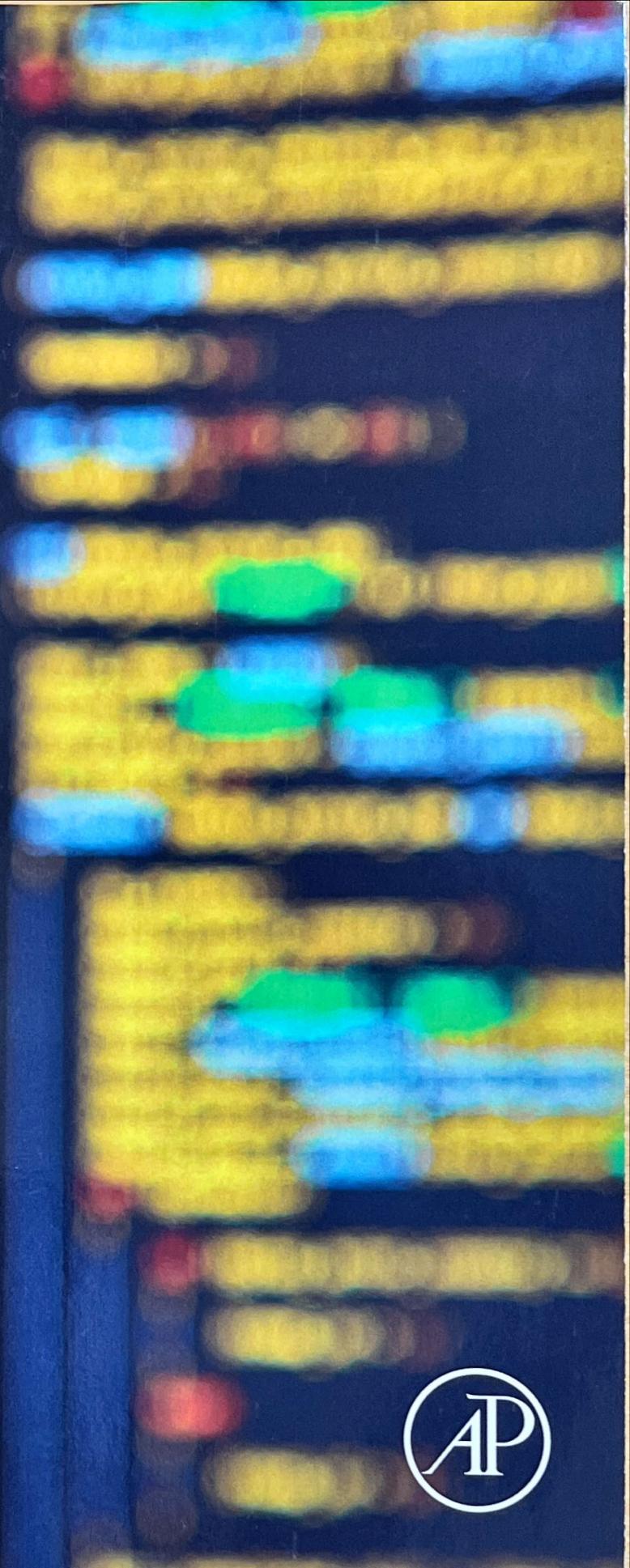




# PYTHON PROGRAMMING AND NUMERICAL METHODS

A GUIDE FOR ENGINEERS  
AND SCIENTISTS

Qingkai Kong  
Timmy Siauw  
Alexandre M. Bayen



# Python Programming and Numerical Methods

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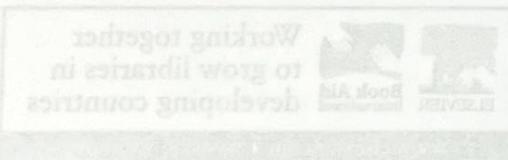
*To my daughter Myriam.*

*As we travel together, we will see lots of interesting people and*



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